The Effects of Supply Shifts on Producers' Surplus: The Case of Inelastic Linear Supply Curves

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Abstract

This paper derives sufficient conditions (in terms of supply and demand elasticities) for producers to gain under different supply shifts when supply and demand are specified to be linear functions and supply is inelastic. It is shown that regardless of the type of supply shift, producers lose whenever the sum of absolute values of supply and demand elasticities is less than one, while they gain when production takes place in the elastic portion of the demand. In all other cases arising from alternative elasticity configurations simple formulas developed in this paper may be used to determine the direction of change in producers surplus.

Keywords: *Supply shift; Producer surplus; linear supply curve*

Introduction

In a series of recent papers, Miller, Rosenblatt and Hushak (1988), Voon and Edwards (1991, 1997); and Elbasha (1997) considered the implication that various functional specifications of supply and demand may have on gross annual research benefits and on producers' welfare under different supply shifts. Among all, linear supply and demand curves received considerably more attention because they have extensively been used for measuring research benefits, due to the simplicity surrounding economic surpluses calculations by means of elementary geometry or algebra. Despite these, Elbasha (1997) suggested that the cases of elastic and inelastic supply functions should be carefully distinguished to each other, particularly in terms of producer surplus measure.[1] He illustrated the importance of this distinction by showing that the use of a single formula for measuring gross annual research benefits for both elastic and inelastic linear supply curves is inappropiate.[1] After making the appropriate adjustment, the estimated gross annual research benefits for the same supply shift, in terms of both type of shift and magnitude, would be smaller under inelastic supply.

Miller, Rosenblatt and Hushak (1988), on the other hand, derived conditions for producers to gain under different supply shifts in the case of linear supply and demand functions. These (sufficient) conditions are related to the slope of the

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supply and demand curves. In particular, they shown that producers always gain if the supply curve shifts outward in a parallel manner, while they benefit from technical change that results in a pivotal shift of the supply curve as long as the slope of the pre-innovation supply curve is greater than the absolute value of the slope of the demand curve. Even though not explicitly stated, these results hold only with elastic supply. Clearly, this is evidence from the definition of linear supply curves (p. 887), used by Miller, Rosenblatt and Hushak (1988) to derive these conditions, as its intercept term is assumed to be strictly positive. Hence, the relevance of these results to agricultural economics profession is limited, as empirical findings ultimately report inelastic supply for most agricultural products in the range 0.1-1.0.

The main objective of this paper to derive sufficient conditions for producers to gain from technical change in the case of linear supply and demand curves, when supply is inelastic.[1] Both parallel and pivotal supply shifts are considered and the conditions derived are stated in terms of supply and demand elasticities. These conditions substantially differ from those derived in the case of elastic linear supply curves. By using the measure of producer surplus suggested by Martin and Alston (1997) and Elbasha (1997) for the case of inelastic linear supply functions, it is shown that farmers may even lose with a parallel shift of the supply curve. In addition, it is apparent that Miller, Rosenblatt, and Hushak's (1988) condition for producers to gain under a pivotal supply shift is only a necessary, but not a sufficient, condition when the case of an inelastic linear supply is considered.

Theoretical Results

Suppose that the supply and the demand curves are specified as $P_s = S(Q) = c + aQ$ and $P_d = D(Q) = d - bQ$ respectively, where $a,b,d > 0$ and $c < 0$. The equilibrium price and quantity are $Q_e = (d-c)/(a+b)$ and $P_e = (cb + ad)/(a+b)$. In this case, the supply curve intercepts the price axis at the negative quadrant and a meaningful representation of producer surplus requires its definition over only the positive price domain (Martin and Alston, 1997; Elbasha, 1997).[2] Thus, producer surplus (PS) is given by the area $P_eAQ_0O = P_eAB - OBQ_0$ (Figure 1), where $Q_0 = -c/a$. In algebraic terms, PS may be written as:

$$PS = \frac{1}{2}\left(\frac{a(d-c)^2}{(a+b)^2} - \frac{c^2}{a}\right) \tag{1}$$

For small changes in the intercept and the slope coefficients of the supply curve, their impact on PS may be evaluated through $\partial PS/\partial c$ and $\partial PS/\partial a$.

First, consider the case of a parallel supply shift which lowers the intercept term. The change in PS is given as:

$$\frac{\partial PS}{\partial c} = \frac{-a(d-c)}{(a+b)^2} - \frac{c}{a} = \left(\frac{-c}{a(a+b)}\right)\left(\frac{P_e}{Q_e}\right)\left(\frac{1}{\varepsilon^d(1-\varepsilon^s)}\right)\left(\varepsilon^s - \varepsilon^d - 1\right) \tag{2}$$

where $\varepsilon^s = (1/a)(P_e / Q_e)$ and $\varepsilon^d = -(1/b)(P_e / Q_e)$ are respectively the elasticity of supply and demand. PS decreases (increases) as c decreases if $\varepsilon^s - \varepsilon^d$ is less (greater) than one. This gives the following basic result:

Result 1: With inelastic linear supply curves, producer surplus decreases, remains unchanged or increases with a parallel supply shift as the sum of absolute values of supply and demand elasticities at the pre-innovation equilibrium is less, equal or greater than one.

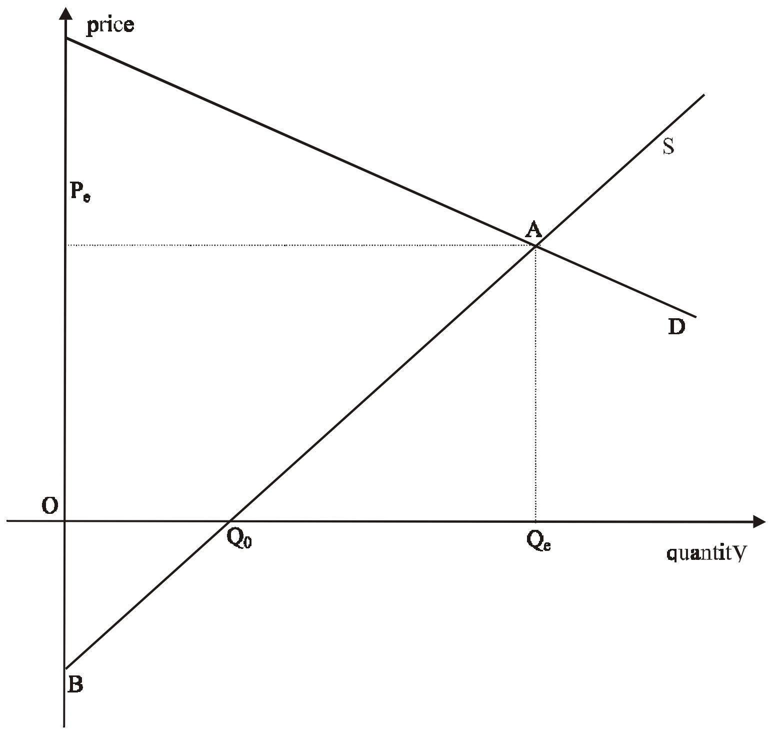

Figure 1. Producers Surplus with Inelastic Linear Supply Curve

This implies that producers gain when pre-innovation equilibrium takes place in the elastic portion of demand, but they may gain or lose if equilibrium is in the inelastic portion depending on the magnitude of supply elasticity. Moreover, producers gain (lose) as the absolute value of both ε^s and ε^d are greater (less) than 0.5. Finally, as $0 < \varepsilon^s < 0.5$ and $0.5 < |\varepsilon^d| < 1$ or as $0.5 < \varepsilon^s < 1$ and $|\varepsilon^d| < 0.5$, producers may gain or loss depending on the relative elasticity magnitudes.

Second, in the case of a pivotal supply shift, which results in a lower slope coefficient, the change in PS is given as:

$$\frac{\partial PS}{\partial a} = \frac{1}{2}\left(\frac{(d-c)^2}{(a+b)^2} - \frac{2a(d-c)^2}{(a+b)^3} + \frac{c^2}{a^2}\right) =$$
$$= \left(\frac{Q_0^2}{a+b}\right)\left(\frac{P_e}{Q_e}\right)\left(\frac{1}{\varepsilon^d\left(1-\varepsilon^s\right)^2}\right)\left(1 - \frac{\left(\varepsilon^d - \varepsilon^s\right)\left(\varepsilon^s - 2\right)}{2}\right) \tag{3}$$

PS decreases (increases) as a decreases if $(\varepsilon^d - \varepsilon^s)(\varepsilon^s - 2)$ is greater (less) than two. This gives the following basic result:

Result 2: With inelastic linear supply curves, producer surplus decreases, remains unchanged, or increases with a pivotal supply shift as the sum of absolute values of supply and demand elasticities at the pre-innovation equilibrium is less, equal, or greater than $2/(2-\varepsilon^s)$.

Given that $0 < \varepsilon^s < 1$, the term $2/(2-\varepsilon^s)$ takes values in the interval [1,2]. Producer surplus decreases when the sum of absolute values of supply and demand elasticities takes a value smaller than 1, while it tends to increase when $\varepsilon^s - \varepsilon^d$ is greater than 2. This implies that producers are harmed from technical change, resulting in a pivotal supply shift, if $\varepsilon^s - \varepsilon^d$ is less than one, while they benefit if pre-innovation equilibrium takes place far in the elastic portion of the demand. But they may also gain if equilibrium is in the inelastic portion of the demand curve, depending upon the magnitude of supply elasticity. The likelihood for this to be true increases as the elasticity of supply approach to unity.

Finally, it can be shown that the (sufficient) condition, derived by that Miller, Rosenblatt, and Hushak (1988), for producers to gain from technical change in the case of elastic linear supply curves, consists only a necessary condition in the case of inelastic linear supply curves. This is evident by rewriting (3) as follows:

$$\frac{\partial PS}{\partial a} = \frac{1}{2}\left(\frac{(d-c)^2}{(a+b)^3}(b-a) + \frac{c^2}{a^2}\right) \tag{4}$$

It is clear that producers may lose for a pivotal supply shift as $b \geq a$, the slope of the demand curve at the pre-innovation equilibrium is greater or equal to the slope of the supply curve.

An Illustrative Example

A qualitative implementation of the above results is provided next by considering the hypothetical example used by Voon and Edwards (1991) and Elbasha (1997). These results are summarized in Table 1. Even though Voon and Edwards (1991) and Elbasha (1997) did not consider the case of a parallel shift and a comparison is not possible in this case, some qualitative results are presented in Table 1 for illustration purposes only. According to our Result 1 and given the alternative elasticity configurations, producers are expected to lose in five out of nine cases. In one case producer surplus is expected to remain unchanged, while in three out of nine cases producers gain.

When the case of a pivotal supply shift is considered, our Result 2 suggests that producers are expected to lose in six out nine cases and to gain in the rest of them (see Table 1). Elbasha's (1997) quantitative findings confirm our theoretical results for all cases considered. This can easily be seen by comparing the signs for producer surplus changes reported on our Table 1 with those on Table 1 in Elbasha (1997, p. 1365). In contrast, Voon and Edwards (1991) quantitative findings contradict with our results in three cases, corresponding to the following pairs of demand and supply elasticities: (-0.01, 0.02), (-0.25, 0.26), and (-0.25, 0.50). For all three, they found a non-negative change in producer surplus, but the sufficient condition for such a change, as stated in Result 2, is not fulfilled in either case. Definitely, this contradiction is related to the formula used to calculated producer surplus when the supply curve is inelastic linear.

Table 1. Changes in Producers' Surplus under Different Elasticity Configurations and Supply Shifts

			ε^d			
	-0.01		-0.25		-1.00	
ε^s	*Parallel*	*Pivotal*	*Parallel*	*Pivotal*	*Parallel*	*Pivotal*
0.01	(-)	(-)	(-)	(-)	(+)	(+)
0.25	(-)	(-)	(-)	(-)	(+)	(+)
0.75	(-)	(-)	0	(-)	(+)	(+)

Note: A negative (positive) sign indicates a decrease (increase) in producers' surplus, while a zero indicates that producers' surplus remains unchanged.

Concluding Remarks

This paper derives sufficient conditions for producers to gain under different supply shifts when supply and demand are specified to be linear functions and the supply is inelastic. These conditions are stated in terms of supply and demand elasticities. As a practical guide, notice that producers lose, regardless of the type of supply shift, whenever the sum of absolute values of supply and demand elasticities is less than one, while they gain when production takes place in the elastic portion of the demand. In all other cases, arising from alternative elasticity configurations, simple formulas developed in this paper may be used to determine the direction of change in producer surplus.

This paper shows that there can not exist a single set of restrictions for farmers to benefit from technical change for both elastic and inelastic linear supply curves. In contrast, a different set of restrictions arise in each case because a different formula is used to measure producer surplus. Thus, the results of the present study should be coupled with those derived previously by Miller, Rosenblatt and Hushak (1988) to provide a complete set of restrictions for the case of linear supply and demand specification. Even though both are stated in terms of supply and demand elasticities around the pre-equilibrium point, *a priori* estimates of supply elasticities available for each study case should be used at the outset to choose the most appropriate set of restrictions.

In the light of recent findings that gross annual research benefits, calculated under the scenario of linear demand and supply schedules, consist the lowest bound when supply is inelastic (Elhasha, 1997; Voon and Edwards, 1991, 1997), the results of the present study can be found useful to policy makers, public administrators and farm groups. For agriculture in particular, where most of supply elasticities are ultimately reported to be less than one, such estimates of gross annual research benefits are relatively more valuable. Moreover, public administrators and farmers organisations, in the case that finance public R&D, may be provided with a good indication of whether producers are going to gain or lose from a particular research project, as the results of this study predetermine the nature of the change in producers surplus independently of the empirical data. Also, they may help them to achieve a reallocation of public funds towards the highest possible social returns from research, which in any case are greater when producers do not lose.

Notes

1 A detailed discussion of issues surrounding the appropriate calculation of producer surplus with elastic and inelastic linear supply functions can be found in Martin and Alston (1997), Elbasha (1997), and Voon and Edwards (1997).

2 By using a single formula (corresponding actually to elastic linear supply curves) Voon and Edwards (1991) overestimated substantially gross annual research benefits, whenever supply was inelastic. The size of overestimation is larger in the case of a parallel rather than a pivotal shift (Elbasha, 1997).

3 The results derived in this paper are applicable only in a partial equilibrium framework.

4 Most of previous studies on measuring gross annual research benefits with inelastic linear supply curves in order to avert this problem have kinked the supply curve at the original quantity. By doing so, however, a poor estimate of the proportionate cost reduction is implied, which gives rise to overestimated returns to research (Alston, Norton and Pardey, 1995, pp. 60-61).

5 According to Wohlgenant (1997), (1) would be appropriate for measuring PS based on linear industry supply and demand curves only if the analysis is restricted either to short-run, where entry is prohibited, or when all firms are intramarginal.

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